Technical Comments

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Comment on "Optimal Feedback Slewing of Flexible Spacecraft"

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IN Ref. 1, Breakwell formulates optimal controls which minimize

$$J=\frac{1}{2}\int_0^{t_f}\left(X^TAX+U^TBU\right)dt \tag{1}$$

subject to the linear autonomous state differential equation

$$\dot{X}=FX+GU \tag{2}$$

with

$$X(0)\text{ and }X(t_f)\text{ prescribed} \tag{3}$$

The resulting optimal control is determined therein from

$$U=-B^{-1}G^T\lambda \tag{4}$$

with

$$\begin{Bmatrix}\dot{X}\\ \dot{\lambda}\end{Bmatrix}=\begin{bmatrix}F & -GB^{-1}G^T\\ -A & -F^T\end{bmatrix}\begin{Bmatrix}X\\ \lambda\end{Bmatrix} \tag{5}$$

Breakwell considers the slewing of a flexible spacecraft whose dynamics can be approximated by Eq. (2). In his conclusions, he discusses two problems encountered previously by the author in Refs. 2 and 3. These are:

1) The fact that the differential equation(s) are stiff and often subject to numerical difficulties if one employs eigenvalue routines or attempts to numerically integrate the associated state transition matrix.

2) The fact that significant vibrational energy is imparted to the higher modes, even though n of them can be arrested upon arrival at time t_f (which is less comforting in the presence of modeling and measurement errors).

Regarding the first problem, Ref. 4 documents an attractive way to obtain high-precision state transition matrices using diagonal Pade approximations in conjunction with the matrix exponential identity

$$e^{Ct}=(e^{Ct/2^n})^{2^n} \tag{6}$$

In Ref. 3, it was applied to essentially the same problem addressed in the present discussion. (For maneuvering of a flexible spacecraft, up to ten modes were considered; the state transition matrix was typically calculated with ten-digit precision.)

Regarding the second problem, the fundamental source of the high-frequency excitation of the vibratory degrees of freedom is the jump discontinuity (initially and finally) in the control torques determined from Eq. (4) (see Figs. 2-7 of Ref. 1). This jump discontinuity can be eliminated easily with a modest modification of the formulation. If, instead of minimizing Eq. (1), we choose the index

$$J=\frac{1}{2}\int_0^{t_f}\left[X^TAX+U^TB_0U+\dot{U}^TB_1\dot{U}+\ddot{U}^TB_2\ddot{U}\right]dt \tag{7}$$

where the overdot indicates a time derivative. Then we still have Eq. (5) but, in lieu of the *algebraic* equation (4), the optimal control satisfied the *differential* equation

$$\frac{d^4U}{dt^4}=B_2^{-1}\left\{-G^T\lambda-B_0U+B_1\frac{d^2U}{dt^2}\right\} \tag{8}$$

and we are free to impose the four *control boundary conditions*

$$U(0)=0 \quad U(t_f)=0 \quad \frac{dU}{dt}(0)=0 \quad \frac{dU}{dt}(t_f)=0 \tag{9}$$

Equations (9) require that the controls be turned on and off smoothly. The maneuvers resulting from Eq. (8) typically require a bit more energy than those using Eq. (4), but the amplitude of the vibrations will typically be reduced by an order of magnitude.

Numerical experience suggests choosing the weight B_2 large enough to dominate the calculations; all of the B_i should be positive-definite-symmetric.

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Comment on "Optimal Control via Mathematical Programming"

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IN Ref. 1, Sheela and Ramamoorthy discuss the numerical solution of a minimum-time low-thrust orbit transfer problem, using a form of the Ritz method.[2,3] The optimized transfer times which they obtain vary over a wide range as the orders of certain series expansions and the time intervals over

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which the expansions are used are varied. In view of the fact that the transfer times, all for the same physical problem, range from 1.33 to 5.83 dimensionless time units (2π such units equals one year), some of the solutions must be quite inaccurate.

This low-thrust orbit transfer problem, from the Earth's heliocentric orbit to the orbit of Mars, idealized as circular and coplanar, with specific values of dimensionless thrust and mass flow rate, has been solved many times previously, using a variety of numerical optimization algorithms.[4-15] Indeed, this problem has served as a standard problem for making comparative assessments of optimization algorithms. The minimum transfer time was shown in early investigations to be 3.32 dimensionless time units (193 days). This minimum transfer time has been corroborated many times subsequently. Only two of the eight transfer times tabulated in Ref. 1 are within 10% of this value. Thus, the optimization algorithm in Ref. 1 does not appear to be performing very well.

Sheela and Ramamoorthy comment that their longest minimum transfer time (338 days), which corresponds to the smallest computed error in satisfying the state differential equations, is quite close to the actual flight time of the Viking-2 spacecraft, from Earth to Mars. They suggest that this close agreement confirms the validity of their 338-day minimum-time solution. The Viking mission involved a ballistic heliocentric transfer, however, rather than a continuous low-thrust transfer. There is no reason to expect the minimum transfer time for a continuously thrusting spacecraft traveling between idealized orbits to match the transfer time for an actual ballistic mission, in which payload delivered into orbit about Mars was maximized, subject to various constraints. For a continuous-thrust transfer, the transfer time is strongly related to the vehicle thrust acceleration. Indeed, the minimum continuous-thrust transfer time can be made arbitrarily short if the thrust acceleration is made sufficiently large and arbitrarily long if the thrust acceleration is made sufficiently small. Thus, any close agreement between a continuous-thrust transfer time obtained in Ref. 1 and the actual Viking-2 transfer time is purely coincidental.

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Reply by Authors to L.J. Wood

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AT the outset we would like to thank Dr. Wood for his comments on our paper.

As can be seen in Table 1 of Ref. 1, the variations in the transfer time t_f for various values of N are less pronounced for the case of $M=4$ than for those of $M=3$. This clearly shows that if the number of functions used for the approximation is increased, the range of variation of t_f will be less. In fact, this number, say M^*, for which the range of variation of t_f for different values of N does not differ should be obtained by the null hypothesis or some other statistical test, and this is not done in the paper.

Regarding the next point raised by Dr. Wood, it is true that the minimum transfer time for this idealized modeling of the orbit transfer problem has been around 3.32 dimensionless units. As for the results tabulated in Ref. 1, the standard deviation of the transfer times taken over different values of N with 3.32 as a mean for $M=3$ and $M=4$ are 1.611 and 0.587, respectively. This clearly shows the convergence trend and that if M is increased further to M^*, defined above, the standard deviation would be still smaller. Thus, it is not true that the algorithm is not performing well.

Finally, we agree with Dr. Wood that the close agreement of our solution for $M=3$ and $N=4$ of Table 1 with that of the actual transfer time for Viking-2 is purely coincidental. In fact, we did not know at the time of our work that the cost criterion for optimization is the payload instead of block time.

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